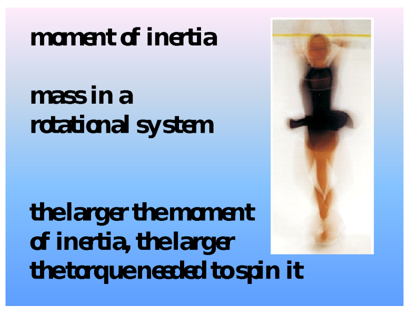
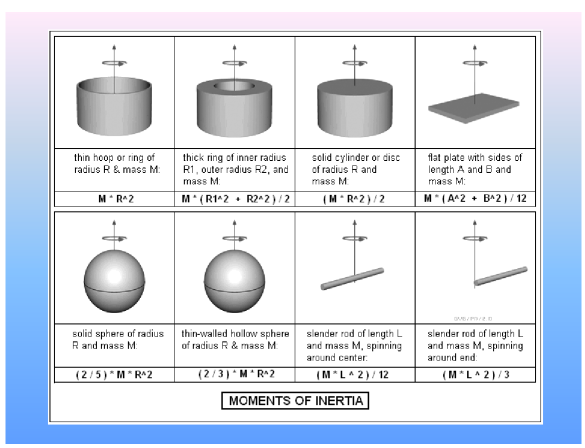
MOMENT of INERTIA / ROTATIONAL KINETIC ENERGY

Mass in a Rotational System / Torque in a Rotational System

There are a variety of wheels available for our electric go-karts. By tradition, we tend to use 20" bicycle or moped wheels. But, do all the schools generally use them because they are the best, or is it because of other factors; ease of access, braking ability, and cost?

Things to think about: Sure, smaller diameter tires can weigh less, but they have to spin faster to match the surface speed of a larger wheel, and this might require more energy and torque depending upon the weight of the wheel. A larger wheel has a harder time resisting the "taco" situation—bending when traveling a corner—whereas a small tire is stiffer. So, your job is to find out which wheel is *actually* better. Which system requires the least amount of energy and torque to revolve at speed? Note: *I will say we use 20" systems because there are good tires available that are light, have sufficient grip, last long, and have low rolling resistance, but this has nothing to do with its moment of inertia and torque.*

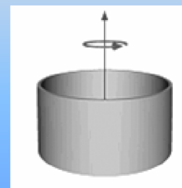




Different shapes require more or less energy to spin at speed. These are common shapes and their associated formulas to calculate for moment of inertia.

MOMENT of INERTIA

$$\mathbf{I = m \times r^2}$$



This shape is most similar to the common wheel, and the moment of inertia formula assigned to it.

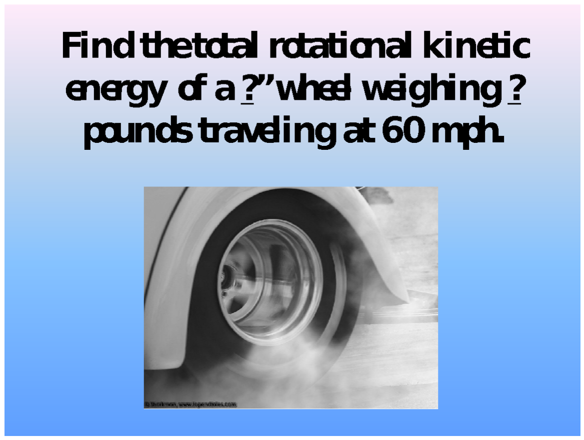
$I = m \times r^2$

$I = (\_\_\_ \text{ lb.} / 32 \text{ ft./sec.}^2) \times (\_\_\_ \text{ ft.})^2$

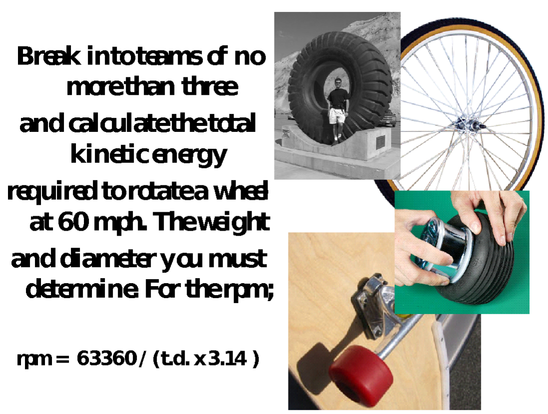
$I = \_\_\_ \text{ lb./ft.sec.}^2 \times \_\_\_ \text{ ft.}^2$

$I = \_\_\_ \text{ lb.ft.sec.}^2$

ROTATIONAL KINETIC ENERGY



You already know how much each wheel weighs, but now you need to calculate the rpm of each wheel with a surface speed of 60 mph. The simplified formula is in the next slide.



rpm = 63,360 ÷ (tire diameter x 3.14)

Rotational Kinetic Energy Formula: $E_K = 1/2 \times I \times \omega^2$
The next step is to convert wheel rpm into radians per second (ω) as performed below.

Using this formula; $E_k = ½ \times I \times \omega^2$, solve for "$\omega$"

ω = (___rev./min. x 1 min./60 sec. x 6.28 rad./

Then, substitute your "I" into the equation.

The last step is to plug in all the variables, the moment of inertia (I) and radians per second (ω), into the formula below and yield a result... now you truly know which wheel requires the most amount of energy to rotate at speed.

$E_k = ½ \times I \times \omega^2$

E_k = ½ x ___ lb.ft./sec.^{2} x ___ rad.^{2}/sec.^{2}

E_k = ___ lb.ft.

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MOMENT of INERTIA

$$I = mr^2$$

given: wheel weight (9.5 lbs.)
wheel diameter (20.75 inches)

$$I = (9.5 \text{ lbs.} / 32 \text{ ft./sec.}^2)\ (0.865 \text{ ft.})^2$$
$$I = (0.296875 \text{ lbs.sec.}^2)\ (0.748225 \text{ ft.}^2)$$
$$I = 0.22129297 \text{ lbs.sec.}^2\text{ft.}^2$$
$$I = 0.22129297 \text{ Joules}$$
$$I = (0.22129297 \text{ Joules})\ (.7375625 \text{ lbs.ft.} / \text{Joule})$$
$$\mathbf{I = 0.163217396 \text{ lbs.ft.}}$$

ROTATIONAL KINETIC ENERGY

$$E_K = 1/2 \times I \times \omega^2$$

given: wheel rpm at 42 mph (680.7 rpm.)
I = 0.163217396 lbs.ft.

((convert rpm into radians))

$$\omega = (680.7 \text{ rev./min.})\ (1 \text{ min.}/60 \text{ sec.})\ (6.28 \text{ rad.})$$
$$\omega = 71.247 \text{ rad./sec.}$$

$$E_K = (1/2)\ (0.163217396 \text{ lbs.ft.})\ (71.247 \text{ rad./sec.})^2$$
$$E_K = (1/2)\ (0.163217396 \text{ lbs.ft.})\ (5076.135009 \text{ rad.}^2/\text{sec.}^2)$$
$$\mathbf{E_K = 414.256769 \text{ lbs.ft.}} \text{ (per wheel)}$$

What the answer means:

The rotational force built up in the wheel of 414 lbs.ft. **per** wheel is the amount of torque required to **STOP** the wheel instantaneously from rotating in middle-air.

This ignores all other factors in the vehicle including aerodynamics of the wheel, traction of the wheel, and the momentum of the vehicle.

It is fair to say that the more energy required to stop a wheel from rotating, the more energy required to bring a wheel up to speed.

At speed, a smaller wheel must spin faster than a larger diameter wheel. Depending upon weight and rpm, it is difficult to know “exactly” how much energy is required to bring a wheel up to speed. Therefore… math!